Sophomore Physics Laboratory (PH005/105)

Analog Electronics
Diodes and Transistors

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(Revision December 2012)
Chapter 4
Diodes and Transistors

4.1 Introduction

In this chapter we will analyze two new electronic devices, the semiconductor diode and the bipolar junction transistor (BJT). For a better understanding of their behavior and characteristics, we will also introduce some basic applications.

Unfortunately, there will be no time to study the quite complex physics of semiconductors, and especially the conduction mechanism, which substantially differs from that of metals. The interested student should look for a course and books on solid state physics.

It is important to notice that to quickly grab how the BJT device works, it is fundamental to acquire a clear understanding of the semiconductor diode’s behavior.

4.2 The Semiconductor Junction (Diode)

The semiconductor junction or semiconductor diode is a device which shows non-linear behavior due to its peculiar conduction mechanism.

In fact, if $I_D$ and $V_D$ are the current and the voltage difference across the junction, we will have

$$I_D(V_D) = I_0(e^{-\frac{qV_D}{k_BT}} - 1), \quad (4.1)$$

where $I_0$ is the reverse saturation current, $k_B = 1.3807 \cdot 10^{-23}$ J/K, the
Boltzmann constant, $T$ the absolute temperature, $q = -1.60219 \cdot 10^{-19}$ C, the electron charge, and $\eta$ a dimensionless parameter which depends on the diode type. Considering that the ambient temperature is $T \simeq 300K$, we will have $k_B T \simeq 4.14 \cdot 10^{-21}$ J $\simeq 0.026\text{eV}$. For silicon diodes the reverse saturation current $I_0$ is of the order of few tenths of nano-amperes.

Instead of following Ohm’s law, the semiconductor junction follows an exponential law. Deviations from this law are negligible depending on the current magnitude and the diode characteristics.

Figure 4.1 shows standard symbols for a semiconductor diode and the I-V characteristic. The break-down voltage $V_b$ reported in the same figure is the reverse voltage which essentially short circuits the junction (typically between -100V and -50V). This behavior is not accounted in equation (4.1), and is generated by the so called avalanche multiplication mechanism and the Zener mechanism$^1$.

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$^1$The thermally generated carriers accelerated by the electric field have enough energy to disrupt the electrons bond of the crystal atoms producing new carriers (electron-holes pairs). The new and accelerated pairs generate new carriers producing an avalanche of carriers, and indeed a break-down current.

A sufficient strong electric field can also disrupt electrons bonds creating an electron-hole reverse current. This effect is called Zener Breakdown mechanism.
4.2. THE SEMICONDUCTOR JUNCTION (DIODE)

A simplified model of the junction diode is that of a perfect switch, i.e.

\[ I_D(V) = \begin{cases} \infty & V \geq V_{on} \\
0 & V < V_{on} \end{cases} \]

where \( V_{on} \) is the diode turn-on voltage or cut-in voltage, which depends on the junction type and on the current magnitude. For current up to \( I_D \sim 100 \text{ mA} \), silicon diodes have \( V_{on} \approx 0.6\text{V} \), and germanium diodes have \( V_{on} \approx 0.3\text{V} \).

For voltages greater than \( V_{on} \), the diode is a short circuit (current is not limited by the diode) and is said to be forward biased. For smaller values it is an open circuit (current across the diode is zero) and is reverse biased.

### 4.2.1 Zener Diodes

Zener diodes are particular semiconductor diodes with adequate power dissipation to operate in the break-down voltage region. They have a well defined \( V_b \), with values ranging from about few volts to several hundreds volts. Zener diode symbol is shown in Figure 4.2. Approximating the characteristics with a piecewise linear relationship, we have

\[ I_D(V) = \begin{cases} -\infty & V < V_b \\
0 & 0 \leq V < V_{on} \\
+\infty & V \geq V_{on} \end{cases} \]

Often, the break-down curve is virtually vertical so that the previous approximation of the reverse biased region is quite good.
4.2.2 Schottky Diodes

A junction made of a semiconductor and a metal can behave like a semiconductor diode[2]. For example, Lightly doped silicon and aluminum can form a semiconductor junction. Such kind of devices, called Schottky barrier diodes (or simply Schottky diodes), still follow the diodes characteristics (4.1) with usually a lower turn-on voltage \( V_{on} \) and a larger in magnitude reverse saturation current \( I_s \). The symbol for Schottky diode device is shown in Figure 4.2.

4.3 Diode Dynamic Impedance

For linear devices the current is proportional to the applied voltage and for a given frequency the impedance \( (V/I) \) is constant. With non-linear circuits this is not true anymore, but we can generalize the impedance concept introducing the dynamic impedance

\[
R_d = \frac{dV}{dI}
\]

Let’s apply this definition to the diode. Starting from the I-V characteristic equation and neglecting the reverse saturation current, after some algebra we obtain

\[
V_D = -\eta \frac{k_B T}{q} \ln \frac{I_D}{I_0}.
\]

Taking the derivative on both sides we obtain

\[
R_d = -\eta \frac{k_B T}{q} \frac{1}{I_D}
\]

As we can see, the dynamic impedance of the diode depends on the current \( I_D \).

Considering a silicon diode with a typical value of \( \eta = 2 \) at room temperature (\( T = 300 \text{ K} \)), we will have

\[
R_d(I_D) \approx \frac{5.2 \cdot 10^{-2}}{I_D}, \quad I_D = 1\text{mA} \Rightarrow R_d \approx 52\Omega.
\]

For small variations of the current around 1mA, we can assume that the impedance of a forward biased diode with \( \eta = 2 \) is \( \sim 50\Omega \). The
4.4 Practical Circuits

To better understand the behavior of a semiconductor junction, let’s analyze a few typical applications of semiconductor diodes. Some other applications in connection to other components will be studied in the following chapters.

4.4.1 Rectifiers, AC to DC Conversion

The purpose of a rectifier circuit is to convert alternating current into a unidirectional current. This can be achieved using semiconductor diodes. The typical alternating current to direct current converter is a rectifier connected to an active low pass filter with a so called regulator circuit, which smooths the rectifier output and minimizes ripples. The simplest regulator is a capacitor placed in parallel with the rectifier output. Regulators can be easily found in literature (see [1]).

4.4.1.1 Half-Wave Rectifier

The simplest rectifier circuit is the so called half-wave rectifier shown in Figure 4.3.

Using the diode ideal characteristic, it is quite straightforward to predict the voltage difference across the the resistor $R_L$. In fact, when the sinusoidal signal is positive, it will forward bias the diode and we will
Figure 4.4: Voltage difference across the load connected to the half-wave rectifier output.

have a voltage drop across the resistor $V_L = RI$. For the negative half cycle, because the diode is reverse biased $V_L$ must be zero.

Considering the diode threshold voltage $V_0$, and the diode resistance $R_f$ during the positive half cycle we will have

$$V_L = \frac{R_L}{R_f + R_L}(V_s - V_0),$$

and if

$$R_L \gg R_f \implies V_L \approx (V_s - V_0).$$

During the negative half cycle we will have

$$V_L = \frac{R_L}{R_r + R_L}V_s,$$

and if

$$R_L \ll R_r \implies V_L \approx \frac{R_L}{R_r}V_s \approx 0.$$

The main disadvantage of this circuit is the very poor efficiency (less than 50% of current is rectified). In fact, instead of rectifying the entire signal the circuit chops the negative half cycle out (see Figure 4.4).
4.4. PRACTICAL CIRCUITS

4.4.1.2 Full-Wave Rectifier Bridge

The Full-Wave rectifier bridge (see Figure 4.5), a more efficient way of rectifying an AC current, uses four arranged diodes in the so-called bridge configuration. To understand the circuit “logic”, let’s consider the two possible states of the nodes A and B shown in Figure 4.5.

- When the node A is positive (B negative) the diodes $D_2$, and $D_3$ are forward biased (i.e. the diodes are a “short circuit”) and $D_1$, and $D_4$ are reverse biased (i.e. the diodes are an “open circuit”). The current flows through the resistor $R_L$ and the node C is positive.

- When the node A is negative (B positive), the diodes $D_1$, and $D_4$ are forward biased (short circuit) and $D_2$, and $D_3$ are reverse biased. The current flows through the resistor $R_L$ and the node C is still positive.

Using the full-wave rectifier we will indeed have the negative half cycle rectified as shown in Figure 4.6.

4.4.2 Voltage Limiter (Diode Clamp)

Diodes can be used to limit the voltage applied to an input as shown in Figure 4.7. Let’s consider the diode $D_1$ connected to $V_{max}$. If $V_i$ exceeds $V_{max} + V_{on}$, the diode is not reverse biased anymore and starts conducting, i.e. the circuit limits the input voltage $V_i$ to $V_{max} + V_{on}$. Analogously, $D_2$ limits the minimum input voltage $V_i$ to $V_{min} + V_o$. The resistor is necessary to limit the current flowing through the diodes. In fact, without the resistor
if we exceed one of the voltage limits an excessive current can destroy the forward biased diode junction. The worst scenario is when the broken diode becomes an open circuit and then the device to protect becomes completely unprotected.

### 4.5 The Bipolar Junction Transistor (BJT)

The bipolar junction transistor is essentially a device formed by two semiconductor junctions which share one semiconductor layer (see Figure 4.8). The common layer is called the base and the two others are the collector.
4.5. THE BIPOLAR JUNCTION TRANSISTOR (BJT)

and emitter. We will have then the emitter-base and the collector-base junctions.

There are two types of BJT: the npn and the pnp transistor. In the pnp transistor the collector and the emitter are p-type and the base is n-type. The npn transistor has a p-type base, and n-type collector and emitter. Standard symbols for both types are shown in Figure 4.9.

Because the two junctions have two possible states (forward or reverse biased), the BJT can have four possible operating modes as shown in the following table

<table>
<thead>
<tr>
<th>Operating Mode</th>
<th>Bias Emitter-Base</th>
<th>Bias Collector-Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward-Active</td>
<td>Forward</td>
<td>Reverse</td>
</tr>
<tr>
<td>Cutoff</td>
<td>Reverse</td>
<td>Reverse</td>
</tr>
<tr>
<td>Saturation</td>
<td>Forward</td>
<td>Forward</td>
</tr>
<tr>
<td>Reverse-Active</td>
<td>Reverse</td>
<td>Forward</td>
</tr>
</tbody>
</table>

**Forward-Active:**

The BJT approximates a current-controlled source of current as explained in section 4.5.2.

**Cutoff:**

Both junctions are reverse biased. Neglecting the reverse saturation current, no current flows through the junctions. This mode, together with the saturation mode, is used to implement the switch device (see section 4.5.5).
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Figure 4.9: Standard circuit symbols for npn and pnp transistors.

**Saturation:**

Both junctions are forward biased, and the current $I_C$ flows from the collector through the emitter.

**Reverse-Active:**

The BJT still approximates a current-controlled source of current, but the amplification factor is usually less than that of the forward-active mode.

### 4.5.1 The Collector Emitter Characteristic

Figure 4.10 shows collector emitter characteristic curves family of a typical npn transistor. Each curve corresponds to a given value of the base current $I_B$, with the base emitter junction forward biased.

The curves have three regions which are called, the \textit{saturation}, \textit{forward-active}, and \textit{breakdown} regions. The break-down region starts for $V_{CE}$ values larger than those shown in the plots.

**Saturation Region**

The saturation region is where the collector emitter voltage difference $V_{CE}$ slightly changes as a function of the collector current $I_C$. For the 2N2222 this regions is where $V_{CE}$ is between 0V to about 0.3V.
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Figure 4.10: Collector Emitter voltage characteristics for the 2N2222 npn transistor. The value above each curve is the corresponding base current $I_B$.

**Forward-Active Region**

The collector current $I_C$ slightly changes as a function of the collector emitter voltage $V_{CE}$. Normally, this region is quite larger than the saturation region. For the 2N2222 it is where $V_{CE}$ is between 0.3V to about 50V.

**Break-Down Region**

This is the region where the $V_{CE}$ doesn’t change and $I_C$ rapidly increases. In this case, the conduction in the junction is produced by the avalanche mechanism. For the 2N2222 this region starts from $V_{CE} > 60V$.

4.5.2 The BJT as a Current-Controlled Current Source (CCCS)

As stated before, the bipolar junction transistor is a device that approximates a current-controlled source of current CCCS (see Figure 4.11). In other words, because its current output $i_o$ is proportional to the current
input $i_i$ we can linearly control $i_o$ by changing $i_i$, i.e.

$$i_o(t) = \beta F i_i(t).$$

If $|\beta F| > 1$ then the BJT is a current amplifier.

As shown in Figure 4.11, once $i_i$ is set $i_o$ must be constant independently of the load $R$ placed at the output. If the voltage across the output $v_o$ changes we don’t expect to see any changes on $i_o$. The curve height simply depends on the current input $i_i$.

This approximation is valid for the so-called small signal model and the low frequency model. Nonlinearities arise for large signals and at high frequency the response cannot be flat.

It is clear from the $V_{CE}$ characteristic that, if we want to use the BJT as CCCS, we have to bias it with a DC voltage to work in the forward-active region.

### 4.5.3 BJT Simplified DC Model

The simplified DC model of the BJT for the forward-active mode is shown in Figure 4.12 with two different arrangements. The first mimics the topology of the BJT symbol, and the second the topology of the CCCS in Figure 4.11. This model is good enough to properly bias the transistor to work as an amplifier.
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Figure 4.12: Simplified DC model for the bipolar junction transistor working in forward-active mode. The two drawings are just two different arrangement of the same circuit.

The current controlled current source represents the $V_{CE}$ characteristic in the forward-active region. The battery in the base emitter circuit represents the voltage across the base-emitter forward biased junction (it could be replaced with a diode). A typical value is $V_{BE} = 0.7V$.

4.5.4 The BJT as an Amplifier

Figure 4.13: The BJT as an AC basic amplifier (left), and same circuit using the forward-active DC model (right).

Left circuit of Figure 4.13 shows the basic configuration of a BJT as a simple current amplifier. Resistors $R_B$ and $R_C$ are chosen to properly bias
and limit the currents across the junctions. The capacitance at the input is necessary to prevent the DC bias from reaching the device connected to the amplifier input. Let’s better analyze how to properly bias the transistor junctions.

### 4.5.4.1 BJT Amplifier Bias

To obtain the largest voltage dynamic range, and considering the $V_{CE}$ characteristic, and neglecting the saturation region, we must have

$$V_{CC} \approx 2V_{CE}.$$  \hspace{1cm} (4.2)

Plugging the forward-active DC model into the amplifier circuit as shown in Figure 4.13, we will have\(^2\)

$$V_{CC} = V_{CE} + R_CI_C,$$

Considering equation 4.2 and the previous equation, the collector resistor value will be

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{V_{CE}}{\beta_FI_B}.$$  

For the base resistor we will have

$$V_{CC} = V_{BE} + R_BI_B,$$

and finally

$$R_B = \frac{2V_{CE} - V_{BE}}{I_B}.$$  

This circuit is not very useful because the junctions bias and the gain depend on $\beta_F$, which is quite often not well known and can easily vary by a factor of two for the same transistor. Moreover, $\beta_F$ is quite sensitive to temperature fluctuations. Anyway, this circuit is pedagogically interesting because of its simplicity.

\(^2\)The repeated index is a common convention used to distinguish between the voltage of the transistor’s connections and the source voltages applied to the transistor connections. In this case between the collector voltage $V_C$ and the source voltage $V_{CC}$.
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Numerical Example

A typical BJT a transistor has $V_{CE}$ between 1V and 10V. Considering the following parameters

\[
\begin{align*}
\beta_F &= 100 \\
V_{CE} &= 5V \\
V_{BE} &= 0.7V \\
I_B &= 80\mu A
\end{align*}
\]

\[
\Rightarrow \quad \begin{align*}
R_C &\approx 625\Omega \\
R_B &\approx 116.25k\Omega \\
V_{CC} &\approx 10V
\end{align*}
\]

4.5.4.2 BJT Amplifier Gain, Input and Output Impedance (Low Frequency Model)

\[\text{Figure 4.14: BJT basic amplifier using the low frequency model (gray box). The parameters } h_{fe} = \beta_F \text{ and } h_{ie} \text{ are provided by the manufacturer}\]

Because the emitter-base junction is forward biased, the input impedance seen from the points $B$ and $E$ is quite low. This consideration with the fact that the BJT approximates a CCCS is sufficient enough to define a model for the BJT transistor response for the low frequency region. Figure 4.14 shows the model applied to the basic amplifier. The resistance $h_{ie}$ is indeed the dynamic input impedance of the forward biased emitter-base junction.

From Figure 4.14 we can easily calculate the amplifier voltage gain, which is

\[
|A_v| = \frac{R_C I_C}{h_{ie} I_B} = h_{fe} \frac{R_C}{h_{ie}} \quad (\beta_F = h_{fe})
\]

Considering that the ideal current source is an open circuit and the ideal voltage source is a short circuit, we will have

\[
R_i \approx R_B || h_{ie}, \quad R_o \approx R_C.
\]
Thermal fluctuations can substantially change the response of the BJT. A way to avoid such kind of behavior is to add a feedback network. Essentially, a feedback network samples the output and sends it back to the input with negative sign minimizing the output fluctuations. For example, if the amplifier gain increases because of a temperature increase, the feedback signal will increase as well reducing the input signal by the amount necessary to keep the gain constant. Feedback networks can create instabilities due to phase delays in the loop (the feedback signal can change sign). It is indeed necessary to satisfy stability criterion to avoid oscillations. A detailed explanation of feedback theory can be found in [2] and [3].

4.5.5 BJT as Switch

Figure 4.15: BJT as a switch

Figure 4.15 shows a npn BJT configured as a switch. In this case, the function of the two resistors $R_B$ and $R_C$ are just to limit the current flowing through the transistor junctions.

The input voltage $v_i$ control the output state of the switch. For sake of simplicity let’s neglect the reverse currents components to study the circuit.

- If $v_i = 0$, The emitter-base junction is reverse biased and no current flows through the circuit. This implies that $v_o \simeq V_{CC}$ and (BJT in cutoff state).
• If $v_i = V$ and supposing that this voltage forward bias both junctions we will have $v_o \simeq 0$ (BJT in saturation).

Let’s consider now the BJT reverse currents.

• If $v_i = 0$, we will have $i_C = I_{CO}$ and $v_o = V_{CC} - I_{CO}R_C$. Because $I_{CO} \sim 1\text{nA}$ $I_{CO}R_C$ is negligible and $v_0 = V_{CC}$.

• If $v_i = V$, then $v_0$ is essentially the voltage drop $V_{BE}$ of the forward biased base-emitter junction $v_0 = 0.7V$.

### 4.5.6 BJT as Diode

![Figure 4.16: BJT as diode.](image)

Figure 4.16 shows the typical configuration used to make a BJT working as simple diode. The emitter-base junction acts as a simple semiconductor diode. Short circuiting the collector-base ensures that the collector-base junction is always reverse biased.

### 4.5.7 Current Mirror

Let’s consider the left circuit shown in Figure 4.17 where $V_{CC}$ forward bias the emitter-base junction. From the KVL obtain

$$I_R = \frac{V_{CC} - V_{BE}}{R}.$$ 

If $V_{CC}$ and $R$ are kept constant ($V_{BE} = 0.7$, typically) then $I_R$ is constant as well. Applying the KCL to node we obtain

$$I_R = I_C + I_B.$$
and considering that \( I_c = \beta I_B \)
we will finally have
\[
I_R = \left(1 + \frac{1}{\beta}\right)I_C. \quad \Rightarrow I_C \simeq I_R.
\]

The collector current \( I_C \) is indeed constant if \( V_{CC} \) and \( R \) are kept constant.

Let’s now consider the circuit on the right-hand side of Figure 4.17. Because of the KVL we will have
\[
V_{BE1} = V_{BE2}
\]

Supposing that the two transistors \( Q_1 \) and \( Q_2 \) are perfectly identical and because they have the same \( V_{BE} \) we must have

\[
I_{C1} = I_{C2}.
\]

We will have indeed that the output \( I_{C2} \) will work as a constant current source.

Let’s analyze the stability of the circuit for a change on the transistor parameter \( \beta \). From the KVL and KCL we have
\[
I_R = \frac{V_{CC} - V_{BE}}{R}
\]
\[
I_R = I_C + 2I_B
\]
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After some simple algebra and considering that $I_C = \beta I_B$ we will have

$$I_C = \frac{\beta}{\beta + 2} \frac{V_{CC} - V_{BE}}{R}$$

Studying the fluctuation of the transistor we will have

$$\frac{\Delta I_C}{I_C} \approx 2 \frac{\Delta \beta}{\beta^2}.$$  

In other words, the stability of this current source due to the fluctuations of the transistor properties are expected to be remarkably good. In fact, if we suppose to have $\beta = 100$ and change of 100% in $\beta$ then

$$\begin{cases} 
\beta &= 100 \\
\Delta \beta &= 100 
\end{cases} \Rightarrow \frac{\Delta I_C}{I_C} \approx 0.02$$

For their simplicity, current mirror are extensively used in ICs design where a constant current source is needed.
Bibliography

[1] ??Find a reference to Regulators??

