Sophomore Physics Laboratory (PH005/105)

Analog Electronics
Basic Op-Amp Applications

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Chapter 5

Basic Op-Amp Applications

5.1 Introduction

In this chapter we will briefly describe some quite useful circuit based on Op-Amp, bipolar junction transistors, and diodes.

5.1.1 Inverting Summing Stage

Figure 5.1: Inverting summing stage using an Op-Amp. The analysis of the circuit is quite easy considering that the node A is a virtual ground.

Figure 5.1 shows the typical configuration of an inverting summing stage using an Op-Amp. Using the virtual ground rule for node A and
Ohm’s law we have

\[ I_n = \frac{V_n}{R}, \quad I = \sum_{n=1}^{N} I_n. \]

Considering that the output voltage \( V_0 \) is

\[ V_0 = -R_f I, \]

we will have

\[ V_0 = A \sum_{n=1}^{N} V_n, \quad A = -\frac{R_f}{R}. \]

### 5.1.2 Basic Instrumentation Amplifier

Instrumentation amplifiers (In-Amp) are designed to have the following characteristics: differential input, very high input impedance, very low output impedance, variable gain, high CMRR, and good thermal stability. Because of those characteristics they are suitable but not restricted to be used as input stages of electronics instruments.

Figure 5.2: Basic instrumentation amplifier circuit.

Figure 5.2 shows a very basic In-Amp circuit made out of three Op-Amps. In this configuration, the two buffers improve the input impedance of the In-Amp, but some of the problems of the differential amplifier are
still present in this circuit, such as common variable gain, and gain thermal stability.

A straightforward improvement is to introduce a variable gain on both amplifiers of the input stage as shown in Figure 5.3(a), but unfortunately, it is quite hard to keep the impedance of the two Op-Amps well matched, and at the same time vary their gain and maintaining a very high CMRR. A clever solution to this problem is shown in Figure 5.3(b). Because of the virtual ground, this configuration is not very different from the previous one but it has the advantage of requiring one resistor to set the gain. In fact, if \( R_3 = R_4 \) then the gain of the Op-Amps \( A_1 \) and \( A_2 \) can be set at the same time adjusting just \( R_G \).

For an exhaustive documentation on instrumentation amplifiers consult [2].

![Figure 5.3](image)

**Figure 5.3:** Improved input stages of the basic instrumentation amplifier.

### 5.1.3 Voltage to Current Converter (Transconductance Amplifier)

A voltage to current converter is an amplifier that produces a current proportional to the input voltage. The constant of proportionality is usually called transconductance. Figure 5.4 shows a Transconductance Op-Amp, which is nothing but a non-inverting Op-Amp scheme.
The current flowing through the impedance $Z_f$ is proportional to the voltage $v_s$. In fact, assuming the Op-Amp has infinite input impedance, we will have

$$i_f(t) = \frac{v_s(t)}{R}.$$

Placing an amperometer in series with a resistor with large resistance as a feedback impedance, we will have a high resistance voltmeter. In other words, the induced perturbation of such circuit will be very small because of the very high impedance of the operational amplifier.

### 5.1.4 Current to Voltage Converter (Transresistance Amplifier)

A current to voltage converter is an amplifier that produces a voltage proportional to the input current. The constant of proportionality is called transimpedance or transresistance, and its units are Ω. Figure 5.5 show a basic configuration for a transimpedance Op-Amp. Due to the virtual ground the current through the shunt resistance is zero, thus the output voltage is the voltage difference across the feedback resistor $R_f$, i.e.

$$v_o(t) = -R_f i_s(t).$$

Photo-multipliers photo-tubes and photodiodes drivers are a typical application for transresistance Op-amps. In fact, quite often the photocurrent produced by those devices need to be amplified and converted into a voltage before being further manipulated.
5.2 Logarithmic Circuits

By combining summing circuits with logarithmic and anti-logarithmic amplifiers we can build analog multipliers and dividers. The circuits presented here implements those non-linear operations just using the exponential current response of the semiconductor junctions. Because of that, they lack on temperature stability and accuracy. In fact, the diode reverse saturation current introduces an offset at the circuits output producing a systematic error. The temperature dependence of the diode exponential response makes the circuit gain to drift with the temperature. Nevertheless these circuits have a pedagogical interest and are the basis for more sophisticated solutions. For improved logarithmic circuits consult [1] chapter 7, and [1] section 16-13 and[3].

Figure 5.6: Elementary logarithmic amplifiers using a diode or an npn BJT.
5.2.1 Logarithmic Amplifier

Figure 5.6 (a) shows an elementary logarithmic amplifier implementation whose output is proportional to the logarithm of the input. Let’s analyze this non-linear amplifier in more detail.

The Op-Amp is mounted as an inverting amplifier, and therefore if \( v_i \) is positive, then \( v_o \) must be negative and the diode is in conduction. The diode characteristics is

\[
i = I_s \left( e^{-qv_o/k_BT} - 1 \right) \simeq I_se^{-qv_o/k_BT}, \quad I_s \ll 1,
\]

where \( q < 0 \) is the electron charge. Considering that

\[
i = \frac{v_i}{R},
\]

after some algebra we finally get

\[
v_o = \frac{k_BT}{-q} \left[ \ln (v_i) - \ln (RI_s) \right].
\]

The constant term \( \ln (RI_s) \) is a systematic error that can be measured and subtracted at the output. It is worth to notice that \( v_i \) must be positive to have the circuit working properly. An easy way to check the circuit is to send a triangular wave to the input and plot \( v_o \) versus \( v_i \).

Because the BJT collector current \( I_c \) versus \( V_{BE} \) is also an exponential curve, we can replace the diode with an npn BJT as shown in Figure 5.6. The advantage of using a transistor as feedback path is that it should provide a larger input dynamic range.

If the circuit with the BJT oscillates at high frequency, a small capacitor in parallel to the transistor should stop the oscillation.

5.2.2 Anti-Logarithmic Amplifier

Figure 5.7 (a) shows an elementary anti-logarithmic amplifier, i.e. the output is proportional to the inverse of logarithm of the input. The current flowing through the diode is

\[
i \simeq I_se^{-qv_i/k_BT}, \quad I_s \ll 1.
\]
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Considering that
\[ v_o = -Ri, \]
thus
\[ v_o \simeq -RIe^{-qv_1/k_BT}. \]

If the input \( v_i \) is negative, we have to reverse the diode’s connection. In case of the circuit of Figure 5.7 (b) we need to replace the pnp BJT with an npn BJT. Same remarks of the logarithmic amplifier about the BJT, applies to this circuit.

Figure 5.7: Elementary anti-logarithmic amplifiers using a diode or an pnp BJT.

Figure 5.8: Elementary analog multiplier implementation using logarithmic and anti-logarithmic amplifiers.
5.2.3 Analog Multiplier

Figure 5.8 shows an elementary analog multiplier based on a two log one anti-log and one adder circuits. For more details about the circuit see [1] section 7-4 and [1] section 16-13.

![Diagram of an elementary analog multiplier](image)

Figure 5.9: Elementary analog divider implementation using logarithmic and anti-logarithmic amplifiers.

5.2.4 Analog Divider

Figure 5.8 shows an elementary logarithmic amplifier based on a two log one anti-log and one adder circuits. For more details about the circuit see [1] section 7-5 and [1] section 16-13.

5.3 Multiple-Feedback Band-Pass Filter

Figure 5.10 shows the so called multiple-feedback bandpass, a quite good scheme for large pass-band filters, i.e. moderate quality factors around 10.

Here is the recipe to get it working. Select the following parameter which define the filter characteristics, i.e. the center angular frequency $\omega_0$, the quality factor $Q$, or the the pass-band interval $(\omega_1, \omega_2)$, and the pass-band gain $A_{pb}$.
5.3. *MULTIPLE-FEEDBACK BAND-PASS FILTER*

![Multiple-feedback band-pass filter diagram](image)

Figure 5.10: Multiple-feedback band-pass filter.

\[
\begin{align*}
\omega_0 &= \sqrt{\omega_2 \omega_1} \\
Q &= \frac{\omega_0}{\omega_2 - \omega_1} \\
A_{pb} &< 2Q^2
\end{align*}
\]

Set the same value \(C\) for the two capacitors and compute the resistance values

\[
\begin{align*}
R_1 &= \frac{Q}{\omega_0 CA_{pb}} \\
R_2 &= \frac{Q}{\omega_0 C (2Q^2 - A_{pb})} \\
R_3 &= \frac{2Q}{\omega_0 C}
\end{align*}
\]

Verify that

\[A_{pb} = \frac{R_3}{2R_1} < 2Q^2\]

See [1] sections 8-4.2, and 8-5.3 for more details.
5.4 Peak and Peak-to-Peak Detectors

The peak detector circuit is shown in Figure 5.11. The basic ideal is to implement an integrator circuit with a memory.

To understand the circuit let’s first short circuit $D_0$ and remove $R$. Then the Op-amp $A_0$ is just a unitary gain voltage follower that charges the capacitor $C$ up to the peak voltage. The function of $D_0$ and of $A_1$ (high input impedance) is to prevent the fast discharge of the capacitor.

Because of $D_0$ the voltage across the capacitor is not the max voltage at the input, and this will create a systematic error at the output $v_o$. Placing a feedback from $v_o$ to $v_i$ will fix the problem. In fact, because $v_+$ must be equal to $v_-$, $A_0$ will compensate for the difference.

Introducing the resistance ($R \approx 100k\Omega$) in the feedback will provide some isolation for $v_o$ when $v_i$ is lower than $v_C$.

The Op-Amp $A_0$ should have a high slew rate (~20 V/µs) to avoid the maximum voltage being limited by the Op-Amp slew rate.

The capacitor doesn’t have to limit the Op-Amp $A_0$ slew rate $S$, i.e.

$$\frac{i_C}{C} \ll \frac{dv}{dt} = S$$

It is worthwhile to notice that if $D_0$ and $D_1$ are reversed the circuit becomes a negative peak detector.

The technology of the hold capacitor $C$ is important in this application. The best choice to reduce leakage is probably polypropylene, and after that polystyrene or Mylar.

Using a positive and a negative peak detector as the input of a differential amplifier stage we can build a peak-to-peak detector (for more details see [1] section 9-1). Some things to check: holding time (a given % drop from the maximum) versus capacitor technology, systematic errors, settling time required for the output to stabilize.

5.5 Zero Crossing Detector

When $v_i$ is positive and because it is connected to the negative input then $v_o$ becomes negative and the diode $D_1$ is forward biased and conducting.
5.6 Analog Comparator

An analog comparator or simply comparator is a circuit with two inputs $v_i$, $v_{ref}$ and one output $v_o$ which fulfills the following characteristic:

$$v_o = \begin{cases} 
V_1, & v_i > v_{ref} \\
V_2, & v_i \leq v_{ref}
\end{cases}$$

An Op-Amp with no feedback behaves like a comparator. In fact, if we apply a voltage $v_i > v_{ref}$, then $V_+ - V_- = v_i - v_{ref} > 0$. Because of the high gain, the Op-Amp will set $v_o$ to its maximum value $+V_{sat}$ which is a value close to the positive voltage of the power supply. If $v_i < v_{ref}$, then $v_o = -V_{sat}$. The magnitude of the saturation voltage are typically about 1V less than the supplies voltages.

Depending on which input we use as voltage reference $v_{ref}$, the Op-Amp can be an inverting or a non inverting analog comparator.

It is worthwhile to notice that the analog comparator circuit is also a 1 bit analog digital converter, which converts voltages to the two levels $V_1$ and $V_2$. 

Figure 5.11: Peak detector circuit.
5.7 Regenerative Comparator (The Schmitt Trigger)

The Regenerative comparator or Schmitt Trigger\(^1\) shown in Figure 5.12 is a comparator circuit with hysteresis.

It is important to notice that the circuit has a positive feedback. With positive feedback, the gain becomes larger than the open loop gain making the comparator to swing faster to one of the saturation levels.

\(^1\)Otto Herbert Schmitt (1913-1998) American scientist considered the inventor of this device, that appeared in an article in 1938 with the name of “thermionic trigger”\([3]\).
Considering the current flowing through \( R_+ \) and \( R_f \), we have

\[
I = \frac{V_1 - V_+}{R_+} = \frac{V_+ - V_o}{R_f}, \quad \Rightarrow \quad V_+ = \frac{V_1 R_f + V_o R_+}{R_f + R_+}.
\]

The output \( V_o \) can have two values, \( \pm V_{sat} \). Consequently, \( V_+ \) will assume just two trip points values

\[
V_+^{(utp)} = \frac{V_1 R_f + V_{sat} R_+}{R_f + R_+}, \quad V_+^{(ltp)} = \frac{V_1 R_f - V_{sat} R_+}{R_f + R_+}
\]

When \( V_i < V_+^{(utp)} \), \( V_o \) is high, and when \( V_i < V_+^{(ltp)} \), \( V_o \) is low.
To set \( V_+ = 0 \) it requires that

\[
V_1 = -\frac{R_+}{R_f} V_o
\]

This circuit is usually used to drive an analog to digital converter (ADC). In fact, jittering of the input signal due to noise which prevents from keeping the output constant, will be eliminated by the hysteresis of the Schmitt trigger (values between the trip points will not affect the output).

See [1] section 11 for more detailed explanations.

**Example1**: \( (V_{sat} = 15V) \)

Supposing we want to have the trip points to be \( V_+ = \pm 1.5V \), if we set \( V_1 = 0 \) then \( R_f = 9R_+ \).

### 5.8 Phase Shifter

A phase shift circuit shown in Figure 5.13, produces a sinusoidal signal at the output \( V_o \) which is equal to the sinusoidal input \( V_i \) with a defined phase shift. The basic idea of this clever circuit is to subtract using an Op-Amp two equal sinusoids one of which is lagging behind because is low pass filtered (the difference of these two same frequency sinusoid is a phase shifted attenuated sinusoid). The Op-Amp provides also the unitary gain.
Using the same method applied to compute the differential Op-Amp stage transfer function, we obtain

\[ H(\omega) = \frac{1 - j\omega RC}{1 + j\omega RC} , \]

The amplitude and phase of the transfer function are therefore

\[ H(\omega) = 1 \quad \text{for any frequency}, \]

\[ \varphi(\omega) = \arctan(-\omega RC) - \arctan(\omega RC) = -2 \arctan(\omega RC) . \]

The phase shift is double the one of a simple low pass filter and therefore can go from 0° down to −180°. Exchanging the potentiometer and the capacitor changes the phase lag to a phase lead.

**Example**

Supposing that we want a phase shift of −90° for a 1 kHz sinusoid with capacitor with a reasonable capacitance value \( C = 10 \text{ nF} \), then

\[ R = \frac{-\tan \left( \frac{\varphi}{2} \right)}{C\omega} = \frac{1}{10^{-8} \cdot 2\pi \cdot 10^3} = 15.915 \text{ k} \Omega . \]

The value of \( R_0 \) can be in the range of few kilohms to tens kilohoms.
5.9 Generalized Impedance Converter (GIC)

The fundamental equation of the GIC, which can be found after tedious calculation, is

\[ Z = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \]

By a careful choice of passive components, one can implement types of impedance with values impossible to attain with standard passive components. This holds true where the Op-Amps behave very closely to ideal Op-Amps and if one lead of \( Z \) is grounded. For example, we can make:

- C to L converter and vice versa,
- LC parallel to LC series and vice versa,
- impedance scaler
- negative resistance
This circuit is also a gyrator, i.e. a two voltage controlled current sources, where the currents have opposite direction. For more details consult [7].

5.9.1 Capacitor to Inductor Converter

If we choose, for example,

\[ Z_1 = Z_2 = R_1, \quad Z_3 = R_3, \quad Z_4 = \frac{1}{j\omega C}, \quad Z_5 = R_5, \]

then the GIC becomes

\[ Z = j\omega R_3 R_5 C. \]

The circuit will behave as an inductor with inductance \( L = R_3 R_5 C \). If we chose large resistance values \( R_3, R_5 \) and reasonably large capacitance values \( C \), we can implement very large inductance values practically impossible to attain using standard inductors.
5.10 Problems Preparatory to the Laboratory

1. Considering the following circuit, determine the voltage output $V_o$ for the following input voltages $V_i = -2V, 1V, 1.5V, 3V$

2. Consider the Schmitt trigger of Figure 5.12.
   (a) If $V_o = -15V$ and $V_+ = 0V$, compute $V_1$.  
   (b) If $V_o = +15V$, and $V_1 = 15V$, compute $V_+$. 

3. Design a Schmitt trigger with two diode clamps and one resistor connected to the output.
   (a) Limit the output $V_o$ from 0 to 5V. 
   (b) Compute the resistance value $R$ necessary to limit the diode current to 10mA.

4. What is the practical maximum and minimum output voltage of the logarithmic amplifier in Figure 5.6?

5. Chose and study at least two circuits to study and design, one from this chapter and one from the next one on active filters.
   New circuits different than those ones proposed in this chapter are also welcome. For a good source of new circuits based on Op-Amps see [1], [4], and [2].
Bibliography


