CALIFORNIA INSTITUTE OF TECHNOLOGY
PHYSICS MATHEMATICS AND ASTRONOMY DIVISION

Freshman Physics Laboratory (PH003)

Measurements and Significant Figures(Draft)

Academic Year 2011-2012

Contents

Appendix A

Measurements and Significant Figures(Draft)

The purpose of this appendix is to explain **how to properly report a measurement and its uncertainty based on the number of significant figures of the uncertainty**. This is a very important task to perform because measurements are meaningless without their associated uncertainty.

The *significant figures* (or *significant digits*) of a number are the digits necessary to specify our knowledge of that number's precision, and therefore they must be carefully chosen to accurately represent that precision. Let's familiarize with first some concepts about numbers representation.

A.1 Scientific Notation

A number n written in scientific notation has the form

$$n = \pm x.xxx \cdot 10^{\pm yyy}$$

where x, y, and z are an arbitrary number of digits.

$$\begin{array}{ll} \pm x.xxx & \text{is the } \textit{mantissa} \\ \pm yyy & \text{is the } \textit{exponent} \end{array}$$

The mantissa is always written with exactly one non-zero digit to the left of the decimal point. For example,

$$n = 12.345 \qquad \Rightarrow \qquad n = 1.2345 \cdot 10^{-1}.$$

A.2 Significant Figures

Definition: *The digits in the mantissa of a number expressed in scientific notation are called significant figures or significant digits. A zero is* ***always*** *significant.*

To determine the number of significant figures, rewrite the number in scientific notation and count the digits in the mantissa. Let's consider some examples.

Number		Scientific Notation	# Significant Figures
134.00	$\Rightarrow$	$1.3400 \cdot 10^2$	5
0.01023	$\Rightarrow$	$1.023 \cdot 10^{-2}$	4
2.3E-4	$\Rightarrow$	$2.3 \cdot 10^{-4}$	2
1.2009	$\Rightarrow$	$1.2009 \cdot 10^0$	5
-0.03201450	$\Rightarrow$	$-3.201450 \cdot 10^2$	7
$20,150$	$\Rightarrow$???	$2.015 \cdot 10^4$	4
	$\Rightarrow$???	$2.0150 \cdot 10^4$	5

The last example is ambiguous because trailing zeros in integer numbers may or may not be significant. Using scientific notation removes this ambiguity.

A.3 Significant Figures in Measurements

Scientific measurements should be reported according to the guidelines of the Bureau International des Poids et Mesures (BIPM).[1] These guidelines may be summarized as follows.

- Every measurement should include its uncertainty and its units.
- A value and its uncertainty should have the same units, exponent, and number of significant figures.
- Scientific notation and SI units and prefixes should be used.

[1]`http://www.bipm.org/en/home/`

- Units are symbols, not abbreviations, and do not need punctuation marks. The official symbol for each unit should be used. (*e.g.*, "3 seconds" is written "3 s" rather than "3 s.," "3 sec.," or "3 sec")

For example, to report c, the speed of light in a vacuum, with its uncertainty we could choose one of the following equivalent options:

$$\begin{array}{ll} c = (2.99792 \pm 0.00030) \cdot 10^8\,\mathrm{m/s} & \text{(extended notation)}, \\ c = (0.299\,792 \pm 0.000\,030)\,\mathrm{Gm/s} & \text{(extended notation with unit prefix)}, \\ c = 2.99792(30) \cdot 10^8\,\mathrm{m/s} & \text{(concise notation)}. \end{array}$$

A.3.1 Significant Figures for Direct Measurements

Let's consider the case of a **single** measurement made directly with an instrument. In this case, the measurement value and its uncertainty must each have enough digits to account for the instrument resolution. Instrument resolution is the maximum error that the instrument gives under the specified conditions, (*e.g.*, measurement range, temperature, humidity, pressure, *etc.*). A statistical analysis and a better characterizattion of the instrument to reduce systematic errors can provide a smaller error estimation.

A.3.1.1 Length Measurement Example

Suppose we use a caliper to measure the outside diameter of a cylinder. The caliper manufacturer has specified a resolution $\Delta x = 0.05$ mm for the caliper. The manufacturer's quoted resolution is usually the maximum error for the instrument when used properly. If a **single** measurement gives 12.15 mm, three proper ways to report the measurement are

$$x = (12.15 \pm 0.05)\,\mathrm{mm}\,,$$

$$x = (1.215 \pm 0.005) \cdot 10^{-2}\,\mathrm{m}\,,$$

and if you really like it

$$x = 1.215(5) \cdot 10^{-2}\,\mathrm{m}\,.$$

If we trust the quoted instrument resolution, it would be misleading to report a measurement as 12.12 mm. This would imply a measurement precision better than the instrument resolution.

Remember that the goal is to accurately estimate the measurement uncertainty. The quoted instrument resolution assumes proper measurement technique. If we are not careful, we may introduce errors that exceed the instrument's quoted accuracy.

A.3.2 Significant Figures for Statistical Measurements

A statistical measurement is made of at least three numbers: the observed measured value, the uncertainty, and the confidence level. The confidence level is the probability, usually in percent, associated to the interval given by the observable value and the uncertainty. In other words, the confidence level is the **probability of a new measurement** to lie inside the mentioned interval. If the confidence level for a statistical measurement is omitted, it is standard to assume the 1σ interval of 68.3%.[2]

Statistical measurements should be reported in extended or concise notation as follows.

$$X = (x.xxx\,xxx \pm y.yyy\,yyy) \text{ units} \qquad [zz.zz\% \text{ Confidence}],$$

$$X = x.xxx\,xxx\,(yyy) \text{ units} \qquad [zz.zz\% \text{ Confidence}].$$

Sometimes the uncertainty interval is asymmetric:

$$X = \left(x.xxx\,xxx\pm^{y.yyy\,yyy}_{w.www\,www}\right) \text{ units} \qquad [zz.zz\% \text{ Confidence}],$$

$$X = x.xxx\,xxx\,\binom{yyy}{www} \text{ units} \qquad [zz.zz\% \text{ Confidence}].$$

How many significant figures should be quoted when reporting a statistical uncertainty? For a direct measurement, there was a clear upper bound from the instrument resolution. For a statistical measurement, there is no such generally-applicable standard. The number of significant figures in a reported result should be chosen to effectively convey the value

[2]68.3% is the probability that a single measurement from a normal distribution lies within one standard deviation of the mean.

and its precision to the reader. Thus, the number should be large enough to accurately reflect the precision, but not so large as to overwhelm the reader with an endless string of digits.[3]

A.3.2.1 Electron Mass Example

For example, the standard value for the electron's mass is the statistical combination of a number of measurements. We therefore report its value as a statistical measurement:

$$\begin{aligned} m_e &= (0.510\,998\,910 \pm 0.000\,000\,013)\ \text{MeV}, \\ m_e &= 0.510\,998\,910\,(13)\ \ \text{MeV}. \end{aligned}$$

In this case, no confidence interval has been specified. We therefore assume the confidence level to be 68.3%, the 1σ interval.

A.3.3 Significant Digits in Calculations

The above discussion concerns *reporting* a measurement result. When performing a calculation, *do not* follow these guidelines for intermediate results—keep as many digits as is practical to avoid rounding errors. At the end, round the final answer to a reasonable number of significant figures for presentation.

A.4 Unit Prefixes

In addition to units, the International System of Units (SI, from the French "Systéme International") specifies a set of standard prefixes that modify the magnitude of a unit.[4] The standard prefixes are listed in Table A.1 By using a prefix, the exponent used to report a measurement in scientific notation

[3]There are varying rules of thumb for choosing an appropriate number of significant figures based on the uncertainty of the measurement. Experimenters who find themselves horrified by the gruesome sight of too many digits in the uncertainty should be tolerant because such "horrific" practice does not seem to inhibit scientific progress.

[4]Detailed information about the SI, including prefixes as well as units, can be obtained from the BIPM website at `http://www.bipm.org/en/si/si_brochure/`.

Magnitude	Symbol	Name	Magnitude	Symbol	Name
10^{-24}	y	yocto	10^{+1}	da	deca
10^{-21}	z	zepto	10^{+2}	h	hecto
10^{-18}	a	atto	10^{+3}	k	kilo
10^{-15}	f	femto	10^{+6}	M	mega
10^{-12}	p	pico	10^{+9}	G	giga
10^{-9}	n	nano	10^{+12}	T	tera
10^{-6}	μ	micro	10^{+15}	P	peta
10^{-3}	m	milli	10^{+18}	E	exa
10^{-2}	c	centi	10^{+21}	Z	zetta
10^{-1}	d	deci	10^{+24}	Y	yotta

Table A.1: SI Prefixes

can be modified but the number of significant figures is unchanged. For example,

$$l = 0.0021\text{m} = 2.1 \cdot 10^{-4}\text{m} = 2.1 \cdot 10^{-1}\text{mm} = 2.1 \cdot 10^{2}\mu\text{m}\,.$$

Although one should generally use standard SI units and prefixes, there are exceptions. In some contexts, non-standard units are more convenient for historical or pragmatic reasons. For example, astronomers customarily measure certain distances in parsecs.[5] An astronomer working with distances to nearby stars would be well-advised to use this non-standard unit. When deciding between an SI and a non-standard unit, one should choose the option that most clearly communicates a result to the intended audience.

[5] A parsec, or parallax-second, is the distance from the Sun to a star that has a parallax of 1 arc second from the Sun. In other words, if we build a right triangle with the Sun on its right angle vertex, the Earth and the start each one on the other vertices, and the angle of the star's vertex is 1 arc second, then the distance star Sun is 1 parsec. The distance Earth Sun is 1 AU (Astronomical Unit) which is the estimated average Earth Sun distance. One parsec is approximately $3.086 \cdot 10^{16}$ m.