

## Direct capture cross sections at low energy

R. D. Williams and S. E. Koonin

*W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125*

(Received 19 December 1980)

We derive simple expressions for the low-energy behavior of direct radiative capture cross sections, particularly those important in solar nucleosynthesis, such as  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ ,  ${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$ , and  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ . Our results for the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  reaction are inconsistent with a measurement by Rolfs *et al.*

[NUCLEAR REACTIONS  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ ,  ${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$ ,  $E < 300$  keV,  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ ,  
 $E < 100$  keV, extrapolated S.]

There recently been much activity concerning reevaluation of the cross sections of certain solar nuclear reactions due to the unexpected low neutrino flux measured by Davis *et al.*<sup>1</sup> One of these reactions,  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ , is very difficult to measure at solar energies but has a direct bearing on the neutrino problem, since the expected neutrino flux from the sun is almost proportional to the low-energy cross section for this reaction.<sup>2</sup> Rolfs *et al.*<sup>3</sup> have remeasured the low-energy S factor (related to the cross section  $\sigma$  by  $S = \sigma E \exp(2\pi\eta)$ , where  $E$  is the entrance channel center-of-mass energy and  $\eta$  the usual Sommerfeld parameter) and claim it has considerably less energy dependence than was found in previous experiments<sup>4</sup> or predicted theoretically.<sup>5</sup> If this were correct, it would substantially alleviate the solar neutrino problem, but would not eliminate it entirely.<sup>2</sup> Our purpose in this paper is to outline relatively simple expressions for the low energy behavior of direct capture cross sections which can be used to extrapolate the high energy ( $E \sim 500$  keV) experimental results for  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  to solar energies ( $E \sim 20$  keV). We find essential agreement with previous work,<sup>4,5</sup> and with recent Caltech measurements,<sup>6</sup> but are in strong disagreement with the data of Ref. 3.

Our treatment is based on the direct capture formalism developed previously,<sup>7</sup> in which the entrance channel and final state are described by the relative motion of  ${}^3\text{He}$  and  ${}^4\text{He}$  clusters. Since only  $E1$  and  $E2$  transitions to the  $p$ -wave bound state from  $s, d$  and  $p, f$  continuum states, respectively, are important at low energy, the S factor for capture to a given final state are

$$S(E1) = k_\gamma^3 (|I_{10}|^2 + |I_{12}|^2), \quad (1)$$

$$S(E2) = \frac{75}{98} k_\gamma^5 (|I_{21}|^2 + \frac{3}{2} |I_{23}|^2),$$

where  $k$  is the photon wave number, and

$$I_{\lambda l} = \int_0^\infty r^2 dr \left[ \psi_f(r) \gamma^\lambda \frac{\varphi_l(r)}{kr} \right] e^{\pi\eta} (2\pi\eta)^{1/2}. \quad (2)$$

Here,  $\varphi_l$  is the continuum wave function describing the  $l$ th partial wave, while  $\psi_f$  is the radial  $p$ -wave bound-state wave function. (For  ${}^7\text{Be}$  the final state has either total spin  $j_f = \frac{3}{2}$  with binding energy relative to the entrance channel of 1.586 MeV, or  $j_f = \frac{1}{2}$  with binding energy 1.157 MeV.) Note that we have omitted many (common) energy independent factors in Eqs. (1) (including a bound-state spectroscopic factor), as it is only the energy dependence of S which concerns us here.

At low energies, it is well known that the integrand in Eq. (2) peaks at radii far outside the range of the strong interactions.<sup>7</sup> Therefore, with good accuracy,  $\psi_f$  may be replaced with the exponentially decaying  $l=1$  Coulomb wave function (related to a Whittaker function). In the same spirit,  $\varphi_l$  may be replaced by a Coulomb wave, modified by the strong interaction phase shift. This we take to correspond to a hard sphere of radius  $r_0$  (see Table I), adjusted to reproduce the elastic scattering,<sup>5</sup> so that

$$\varphi_l(r) = F_l(kr) - \frac{F_l(kr_0)}{G_l(kr_0)} G_l(kr), \quad (3)$$

where  $k$  is the entrance channel wave number and  $F_l$  and  $G_l$  are the usual regular and irregular Coulomb wave functions. Consistent with this description, we limit the integral in Eq. (2) to  $r > r_0$ .

To obtain tractable expressions for the  $I_{\lambda l}$ , it is useful to employ the Bessel function expansions of  $F_l$  and  $G_l$  (Ref. 8) to generate series in powers of  $E$ , each term involving  $E$ -independent radial integrals of Bessel functions, powers, and Whittaker functions. These can be readily evaluated numerically by Laguerre quadrature to ob-

TABLE I. Kinematic parameters and results.

	Binding (MeV)	Branching (%)	$r_0$ (fm)	$a$ (MeV <sup>-1</sup> )	$b$ (MeV <sup>-2</sup> )
<sup>3</sup> He( $\alpha, \gamma$ ) <sup>7</sup> Be					
$j_f = \frac{3}{2}$	1.586	73	2.8 <sup>a</sup>	-0.575	-0.005
$j_f = \frac{1}{2}$	1.157	27			
<sup>3</sup> H( $\alpha, \gamma$ ) <sup>7</sup> Li					
$j_f = \frac{3}{2}$	2.468	73	2.8 <sup>a</sup>	-2.034	-3.709
$j_f = \frac{1}{2}$	1.989	27			
<sup>7</sup> Be( $p, \gamma$ ) <sup>8</sup> B	0.136		4.1 <sup>b</sup>	-2.350 <sup>c</sup>	28.3 <sup>c</sup>

<sup>a</sup> Reference 5.<sup>b</sup> Reference 7.<sup>c</sup> Equation (4) valid only for  $E \leq 100$  keV.

tain the low-energy behavior of  $S$ .

We have considered the reactions <sup>3</sup>H( $\alpha, \gamma$ )<sup>7</sup>Li and <sup>7</sup>Be( $p, \gamma$ )<sup>8</sup>B as well as that of primary interest, <sup>3</sup>He( $\alpha, \gamma$ )<sup>7</sup>Be, and have calculated the low energy logarithmic derivative of  $S$  in the form

$$\frac{1}{S(E)} \frac{dS}{dE} = a + bE. \quad (4)$$

For the  $\alpha$ -capture reactions, where two final states are involved, we have taken the low energy branching ratio to be energy independent and equal to the experimental value.<sup>9</sup> The kinematic parameters for the three reactions and our results for  $a$  and  $b$  are shown in Table I. Our value for  $a$  for <sup>3</sup>He( $\alpha, \gamma$ )<sup>7</sup>Be agrees with more involved numerical calculations<sup>5</sup> and is consistent with

previous<sup>4</sup> and as yet unpublished<sup>6</sup> Caltech results; it is inconsistent with the data of Ref. 3. Our value for  $a$  varies by less than 5% for changes in  $r_0$  of 10%. For <sup>7</sup>Be( $p, \gamma$ )<sup>8</sup>B, we find the previously expected<sup>9</sup> sharp rise in the  $S$  factor below  $E = 100$  keV; at energies greater than this, the expansion implied by (4) does not converge, due to the small binding energy of the final state.

We are grateful to Professor William A. Fowler for suggesting this problem, and have benefitted from discussions with J. L. Osborne, P. D. Parker, and T. A. Tombrello. This work was supported in part by the National Science Foundation (PHY77-21602 and PHY79-23638) and in part by the Alfred P. Sloan Foundation.

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