1 SUSP3DMASS BY HIRO YAMAMOTO

1.1. A simple 3D test mass module in the End to End model

The LIGO End to End model (e2e) is a time domain simulation program developed for the LIGO detector. It is very flexible and can simulate a wide variety of configuration of optics, mechanics and control systems. Each physical device is simulated by a corresponding software object called “primitive module”. A configuration to be simulated is constructed by combining these primitive modules.

The simplest simulation of a single suspended mirror is done by using a simple digital filter with a complex pole pair. It can simulate only the longitudinal degree of freedom by the pendulum motion of the mass hung by a stiff wire. In order to simulate the motion of a suspended mirror by two wires, a module needs to be built based on the dynamics of 3 dimensional objects. The 3d mirror module of e2e is based on the calculation of M. Rakhmanov of U.Florida. This 3D mirror module calculates the position and the orientation of a mirror for a given suspension point and actuation forces on the mirror.

1.2. e2e primitive module “susp3Dmass”

The 3d mirror model is consisted of two stiff wires without violin modes and a mirror of perfect cylindrical shape without internal modes. The mirror surface does not have any wedge angle, but the center of the mass can be different from the geometrical center. The schematic view of the mirror module is shown in Fig. 1.

Figure 1: Single Suspended Mass

The naming of various settings of this module follows the one used in the suspension design documents as much as possible. [ (1) S. Kawamura and J. Hazel. LIGO-T970135 “Small Optics Suspension Final Design (Mechanical System)” , (2) S. Kawamura, J. Hazel and M. Barton.
LIGO-T970158 “Large Optics Suspension Final Design (Mechanical System)”. The default values of these settings are the values of the large optics suspension:

### Table 1: settings and default values

<table>
<thead>
<tr>
<th>name</th>
<th>description</th>
<th>default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>mirror diameter</td>
<td>0.250 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>mirror thickness</td>
<td>0.10 m</td>
</tr>
<tr>
<td>d_yaw</td>
<td>distance between the two suspension points at the upper release point</td>
<td>0.0333 m</td>
</tr>
<tr>
<td>d_attach</td>
<td>horizontal distance between the points where wires touch standoffs.</td>
<td>0.25506 m</td>
</tr>
<tr>
<td>d_pendulum</td>
<td>length of the pendulum</td>
<td>0.450 m</td>
</tr>
<tr>
<td>d_CM</td>
<td>vertical deviation of the center of mass from the center of cylinder</td>
<td>0.0014 m</td>
</tr>
<tr>
<td>d_pitch</td>
<td>height from the horizontal level through the center of cylinder to the wire release points</td>
<td>0.0082 m</td>
</tr>
<tr>
<td>Mass</td>
<td>weight of the mirror</td>
<td>10.30 kg</td>
</tr>
<tr>
<td>OvalInvZ</td>
<td>1 / Q for the longitudinal motion</td>
<td>1e-4</td>
</tr>
<tr>
<td>QvalInvPITCH</td>
<td>1 / Q for the pitch motion</td>
<td>1e-4</td>
</tr>
<tr>
<td>QvalInvYAW</td>
<td>1 / Q for the yaw motion</td>
<td>1e-4</td>
</tr>
</tbody>
</table>

The inputs to the module are

- the suspension point (“suspPt”) location \( (s_1 \hat{1} + s_2 \hat{2})/2 \) and the angle of orientation \( s_1 \hat{1} \rightarrow s_2 \hat{2} \), and
- the force and torque acting on the mirror (“force”).

The output calculated by the module is

- the location and the orientation of the mass (“massPos”).

For the convenience, two coordinate systems are used, optics coordinate system and suspension coordinate system, as shown in Fig. 2. This is the side view of a suspended mirror. The dashed box is the mirror and the dashed line is the wire when they are at rest. The difference of the two coordinates are locations of the origins, and the orientations are the same. The \( z \) axis of the optics coordinate is the laser beam direction. The origin of the \( z \) axis is on the surface of the coated side of the mirror at rest. The directions of \( x \), \( y \) and \( z \) are shown in the same figure. The origin of the suspension coordinate is located at \( (0, d\_pendulum, -\text{Thickness}/2) \) in the optics coordinate.

To specify the suspension point, the suspension coordinate is to be used, and the calculated mirror location is given using the optics coordinate. With this convention, independent of the mirror size
and wire length, the center of the coated side of the mirror at rest is (0,0,0) when the suspension point is (0,0,0).

![Diagram](image)

**Figure 2: Two coordinates**

Practically, the simulated stack top motion is passed to this module as the suspension point motion, and the calculated mirror location can be used to calculate the field interacting with this mirror. The vertical oscillation is not calculated in this module. When the suspension point moves vertically, that change is added to the vertical motion of the mass. In other words, when the suspension point is given as (dx, dy, dz), then the mirror center is offset by (dx,dy) with respect to the beam axis.

The torque should be calculated with respect to the center of mass location (a square box in Fig. 1), which is located at (0, d_CM, -Thickness/2) in the optics coordinate system. Using the forces acting on the four actuators on the mirrors, the force and torque input to the mirror are calculated as follows.

\[
\hat{F} = \sum_{i=1}^{4} \hat{f}_i
\]

\[
\hat{\Omega} = \sum_{i=1}^{4} (\hat{x}_i - \hat{x}_{CM}) \times \hat{f}_i
\]

Here, \( F \) is the force vector and \( \Omega \) is the torque vector. \( f_i \) is the force acting on the i'th actuator and \( x_i \) is the position of the actuator in the optics coordinate system. In the calculation of torque, the coordinates need to be adjusted to take into account of the difference of the geometrical center of the cylinder and the center of the mass. In this equation, \( x_{CM} \) is the location of the center of mass, i.e., (0, d_MC, -Thickness/2).

When all forces are along the z axis, the equation above can be simplified as follows with all other components 0.
\[ F_z = \sum_{i=1}^{4} f_i \]

\[ \Omega_x = \sum_{i=1}^{4} (y_i - d_{CM}) \cdot f_i \]

\[ \Omega_y = -\sum_{i=1}^{4} x_i \cdot f_i \]
1.3. Simulations in e2e

1.3.1. Free swing and coupling of z and pitch motion

Fig. 3 is a setup to simulate a free swing of a mirror. The primitive module named susp3Dmass contains all the dynamics. The initial condition is defined that z position = 0.01 using the setup window. The two small boxes on the left of the susp3Dmass module are the inputs to specify the suspension point motion and actuation force. There is no force acting and there is no motion defined for the suspension point, so this set up is for the simulation of a freely swinging mirror. Fig. 4 shows the result of the run. The top figure is the time evolution of the z position and the pitch value and the bottom figure is the frequency components. For the setting values of this mass, the z motion resonance frequency is 0.75 Hz and the pitch motion resonance frequency is 0.60 Hz.

![Figure 3: free swing setup](image)

![Figure 4: free swing motion](image)
1.3.2. Seismic motion and mirror motion

Fig. 5 is a setup to simulate the suspended mirror motion on a BSC stack. The two boxes named “Seismic motion in one axis” generates time series of the ground motion in one direction, and the independent motion in the z axis (beam direction) and in the x axis (in the detector plane and perpendicular to the beam direction). The motion in the y axis and all rotational noises are neglected. BSC box is composed of 36 transfer functions, and digital filter modules are used to simulate those linear responses.

![Figure 5: seismic noise setup](image)

Fig. 6 is the simulated motions of z, pitch and yaw. Because the rotation in the plane is very small, the yaw motion due to the seismic motion is very small. The yaw value is scaled by $10^{12}$ for the convenience. The top figure is the time motion and the bottom figure is the frequency distribution. Resonance frequencies of each DOF can be observed at 0.75 (z), 0.60 (pitch) and 0.5 (yaw). The peak at around 1.3 Hz is induced by the stack resonance.

![Figure 6: seismic noise motion](image)
1.3.3. Actuators and mirror

Fig. 7 shows how to drive a mirror with the 4 actuator forces. The content of a module “fi2FrcTrq” is shown in the bottom half of Fig. 7. In susp3Dmass, d_CM is defined to be d_CM_value, where d_CM_value is a macro defined to be 0.0014. The same macro name is used in fi2FrcTrq setting code.

In this code, all four actuators are located completely symmetrically, but if an effect of asymmetry is to be studied, modify one of the following lines defining the actuator positions:

```c
/* x - y coordinates of actuators */
x1 = Rmag; y1 = Rmag;
x2 = -Rmag; y2 = Rmag;
x3 = -Rmag; y3 = -Rmag;
x4 = Rmag; y4 = -Rmag;
```

Figure 7: actuators and mirror

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```
1.3.4.  susp3Dmass in FP cavity with LSC

to be added later