Physics 106a, Caltech — 1 October, 2019

Course Organization

Most of the information here is on the course website:
https://labcit.ligo.caltech.edu/∼ajw/ph106
Please read the course policies below!
Acknowledgement: Much of the material for this course (lectures, assignments, solutions) were developed by Prof. Michael Cross; many thanks to him!

Me

Prof: Alan Weinstein
E-mail: ajwcaltech.edu (questions, comments, arrange meeting. . .)
Office: 354a West Bridge, x2166. Office hours: TBD.

Teaching assistants

- Ka Yue "Alvin" Li, kli7 AT caltech.edu, will grade Assignments: 1, 6, and the Final (all of these are tentative, subject to change!)
- Yanlong Shi, yshi3 AT caltech.edu, will grade Assignments: 2, Midterm, 7.
- Chien-Chang "Kyle" Chen, cchen AT caltech.edu, will grade Assignments: 3, 4, 5.

Textbooks for Phys 106a:

Analytical Mechanics by L.N. Hand and J.D. Finch, Cambridge University Press (1998) (on amazon).
Much of the structure, pacing, notation, etc. is taken from this text. It is far from perfect: it doesn't have a review of elementary Newtonian mechanics; it has typos; many people think the explanations are often unclear. Looking at other texts can always help.

Recommended texts: On reserve at Fairchild Library. Use these texts for alternate explanations or for additional problems or examples.

- Classical Mechanics by Goldstein, Poole, and Safko (3rd edition): This is a classic textbook (I was taught the subject using an earlier edition, back in the 70s).
- Classical Mechanics by John Taylor: a nice book, with more review of the basics than Hand and Finch, but slightly less advanced than the level of the class; it will need supplementing with other reading in a few places.
- Mechanics by Landau and Lifshitz: classic but terse.
- Classical Dynamics of Particles and Systems by Thornton and Marion: not as advanced as class text, and does not cover all the material, but good supplement if you find the jump from earlier classes to Ph106 too large.

Topics for Fall Term

1. Review of Newtonian mechanics
2. Variational approach
3. Lagrangian mechanics
4. Constrained dynamics
5. Equilibria and oscillations
6. Central forces: Bound states and scattering
7. Hamiltonian dynamics
8. Small vibrations and normal modes
9. Rotations, rotating coordinate systems, Dynamics of rigid bodies
10. Special relativity

Assignments, exams, grading

Grading:
Approximately 50% problem sets, 15% midterm, 35% final exam.

- Problem sets and exams will be posted below on Friday of each week, due on the following Friday at 5pm in the Physics 106 IN box by the East Bridge mailboxes, and returned to the Physics 106 OUT box by the East Bridge mailboxes.
- Please write as neatly as possible. A human being is trying to read your work well enough to give credit!
- To satisfy FERPA privacy rules: please fill out a FERPA waiver and give it to the Physics administrator. If you don't want to do this, please choose a 6 or 7 digit code and email this to me and the TAs before the first assignment is due. You will use this, and not your name, to label the assignments you hand in. If you have not filled out a FERPA waiver, and if your name, and not just your numerical code, appears on the assignment, you will have to pick it up directly from the TAs.
- Solution sets will be posted on the web. You are strongly encouraged to check your work when it is returned to you.
- Corrections to problems may be posted in the Announcements section above. If you are having trouble with a problem, be sure to check this page to see if a correction has been posted, and feel free to contact me if you think a problem has errors in it or seems overly difficult. Problem sets are essential for mastering the material in this class!
- Midterm and final exams during appropriate weeks of the term (5th and 10th). Both will be take-home and "limited" open-book (only the text and class notes allowed); see Honor Code section below. The final exam will be comprehensive.

Policies

Extensions:

- OFFICIAL policy: Work (the entire problem set) will be accepted up to one week late at 1/2 credit, no credit thereafter. Please put a note at the top of your problem set if it is late. You do not need to contact me or the TAs to turn in a problem set late for 1/2 credit. You must hand in the assignment in one piece (i.e. not a fraction on time for full credit, and the rest late for partial credit).
- Students may request extensions from the corresponding grader (see emails above) a day or more in advance. Extension requests are governed by the honor system.
- One extension (for up to one week) is allowed without question (your silver bullet). Please put a note at the top of your problem set that you are using your silver bullet.
- Extension requests should be accompanied by a good excuse (eg, physical or mental illness).
- You must request the extension (by email to me and the TAs) before the problem set is due. (This is not necessary for your silver bullet).
- Please put late or extension problem sets in the corresponding Ph106 mail box, and email them.
- Late papers make far more work for the graders, who have their own set of pressures and deadlines as graduate students. There is no entitlement to extensions, so please do not be demanding.

Honor Code and Collaboration policy:

- Work is governed by the Honor Code. You may not use sources that contain the answer to a problem or to a very similar problem. If you have come across such material in the past, so much the better; but you shouldn't go back and reference that material when working on the problem set.
- In particular, **do not use solution sets from previous years, or problem/solution books, at any time. Exams and their solutions from past years are not to be used in any fashion.**
- Collaboration is permitted on problem sets, but then you should go off alone and write it up; the work you hand in must be your own, and honestly reflect your own understanding of the material.
- The midterm and final are not collaborative. For these you may consult your own notes (both in-class and any additional notes you take), the text by Hand and Finch, and handouts and solution sets on this website. You may not use other textbooks, the web (except for the current Ph106 website), or any other resources.
- Mathematica or similar software may be used in problem sets for getting past some mathematical chore (if it's to the point of obscuring the physics for you). However, it is usually much better to master the mathematical analysis yourself without help from such software. If you chose to use it anyway, make sure you simplify the result as much as possible, so that it is easy to see what the math is telling you.

- Mathematica or similar software may not be used in exams, unless explicit stated on the exam.
- Please attend class, and section meetings!
- Please ask questions of the TA's and the prof!

Feedback: The prof and the TAs greatly appreciate student feedback; feedback prior to the end-of-term TQFR evaluations lets me modify the class to fit your needs. Talk with us, email us, or use the feedback link on the website.

Lecture 1: Review of Newtonian Mechanics

This lecture reviews some of the basic ideas of Newtonian mechanics, just to get things going. I will go over the concepts rapidly, and leave it up to you to fill in the details if there are parts you are not familiar with or do not remember. Hand and Finch do not have a review chapter, so you can consult your favorite elementary textbook, or *Taylor* Chapters 1 and 3 or *Goldstein, Poole, and Safko* §1.1-2 (different levels of terseness!).

Newton's Laws

1. In the absence of forces, a particle moves in a straight line with constant speed v (or, equivalently, moves with a constant *velocity* $\vec{v}$ where $\vec{v} = v\hat{v}$, with $\hat{v}$ a unit vector giving the direction of the straight line motion).

2. For a particle of mass m, the *acceleration* $\vec{a}$ for an imposed force $\vec{F}$ is given by

$$\vec{F} = m\vec{a} \tag{1}$$

   An equivalent formulation is in terms of the *momentum* $\vec{p} = m\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt} \equiv \dot{\vec{p}} \tag{2}$$

3. If object 1 exerts a force $\vec{F}_{12}$ on object 2, then object 2 exerts a reaction force $\vec{F}_{21}$ on object 1 given by

$$\vec{F}_{21} = -\vec{F}_{12} \tag{3}$$

   or "action and reaction are equal and opposite."

Some points to think/learn more about:

- Think about the genius of Galileo and Newton in coming up with the first law in a familiar terrestrial world where all undriven motion eventually ceases, or the astronomical world studied by scientists where objects move in nearly perfect circles. Historically, accepting the first law implied accepting the possibility of action at a distance (gravity) not just pushes and pulls from direct contact.

- Are the concepts of mass and force defined (qualitatively, quantitatively) outside of Eq. 1? Is the equation a law of a physics or a definition of force?

- Do you know of a simple example where the third law is violated? How do you reconcile this with the accepted "truth" of Newton's laws?

- How and why will relativity and quantum mechanics change all of this? How about gravity, thermodynamics, rotational motion?

The book *Isaac Newton* by *James Gleick* is interesting to read for the struggles Newton had in arriving at the right ideas and words to state the laws of motion.

Other Concepts

The formulation of the laws of motion rely on a number of underlying concepts:

- Many of the concepts in mechanics are only (or best) defined in context (eg, $F = m\vec{a}$). Physics strives for *self-consistency* in concepts and formulas.
- The notions of the framework of space and time. The Newtonian concepts, particularly the idea of absolute time, need to be modified in special relativity.
- An observer system or *reference frame* (e.g. a set of rulers and clocks) for quantifying separations and time intervals.
- Inertial frames: the special set of reference frames ("nonaccelerating frames") for which the laws of motion hold. Different inertial frames may be in relative motion, but only with a constant relative velocity $\vec{V}$. The room where we sit, or do experiments, is only *approximately* an inertial frame.
- Vectors, such as $\vec{a}, \vec{v}, \vec{p}$, which take advantage of the physical principle of the rotational symmetry of space to write the equations in a form independent of coordinate basis.
- Vector equations such as $\vec{F} = m\vec{a}$ are thus coordinate-independent.
- A system of coordinates (Cartesian, polar ...) to evaluate the consequence of the laws of motion.
- Considerations of symmetry. We will see how space-time symmetries such as homogeneity of space, isotropy of space, and time-transation invariance severely restrict the possible motion of physical systems, and lead to *conservation laws*.

An important invariance or symmetry known as *Galilean invariance* is that Newton's laws of motion are unchanged by transforming to a different inertial frame, moving with respect to the original frame by a constant velocity $\vec{V}$. Under such a transformation

$$t \to t' \quad \text{with} \quad t' = t \tag{4}$$

$$\vec{r} \to \vec{r}' \quad \text{with} \quad \vec{r}' = \vec{r} - \vec{V}t \tag{5}$$

$$\vec{v} \to \vec{v}' \quad \text{with} \quad \vec{v}' = \vec{v} - \vec{V} \tag{6}$$

$$\vec{a} \to \vec{a}' \quad \text{with} \quad \vec{a}' = \vec{a} \tag{7}$$

and it is assumed that the force is the same as measured in the two frames (forces are *invariant* under Galilean transformations). Note, of course, that Newton's laws are *not* true in non-inertial (accelerating frames) — such as on the surface of the (rotating) Earth. We will discuss some consequences of this ("virtual" forces, leading to, e.g., hurricanes) in later lectures.

Newton's laws of motion were profoundly important historically in establishing science as a quantitative pursuit, and the concepts introduced still shape how we think about and quantify the everyday world around us. However as science pushed into new regimes, it became clear that concepts introduced by Newton such as force are not the best way to think about small scales (quantum mechanics), very large scales (general relativity, curved space time), or motion at high speeds (special relativity). The alternative formulation of Newtonian physics known as *Lagrangian mechanics* that we will study in detail provides a more direct route to the extensions needed in these regimes.

Further Developments

Conservative forces and energy

Define the *work* done by the external force $\vec{F}$ acting on a particle in going from point 1 to point 2 by the "first spatial integral":

$$W_{12} = \int_1^2 \vec{F} \cdot \vec{ds}. \tag{8}$$

Use Newton's 2nd law to write $\vec{F} = m d\vec{v}/dt$, and replace the line integral by an integral over time using $\vec{ds} = \vec{v} dt$ to find

$$W_{12} = m \int_{t_1}^{t_2} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt = \frac{m}{2} \int_{t_1}^{t_2} \frac{d}{dt}(v^2)\, dt = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2. \tag{9}$$

We call the quantity $\frac{1}{2} m v^2$ the *kinetic energy.*

If W_{12} is *independent* of the path taken from 1 to 2 we say the force is *conservative.* Equivalently, the integral of a conservative force around a *closed loop* is zero (try to convince a bicyclist of this!)

$$\oint \vec{F} \cdot \vec{ds} = 0 \quad \text{or} \quad F = -\vec{\nabla} V(\vec{r}), \tag{10}$$

with $V(\vec{r})$ the *potential energy* (review an elementary mechanics text or the introduction of potential in electrostatics if you are not happy with these steps). For a conservative force

$$\tfrac{1}{2} m v_1^2 + V(\vec{r}_1) = \tfrac{1}{2} m v_2^2 + V(\vec{r}_2) \equiv E_{tot} \tag{11}$$

and the *total energy* (kinetic plus potential) is conserved in the dynamics. Not all familiar forces are conservative: gravity and electrostatic forces are; magnetic forces and "dissipative" forces such as friction are not.

Many particle dynamics, conservation of momentum and angular momentum

For a set of many particles i (not necessarily of the same mass) Newton's second law reads

$$\dot{\vec{p}_i} = \sum_j \vec{F}_{ji} + \vec{F}_i^{(e)} \tag{12}$$

with $\vec{F}_{ji}$ the interparticle forces and $\vec{F}_i^{(e)}$ the forces from external sources. Introducing the *total momentum* $\vec{P} = \sum_i \vec{p}_i$ and summing over all particles i gives

$$\dot{\vec{P}} = \sum_{ij} \vec{F}_{ji} + \sum_i \vec{F}_i^{(e)}, \tag{13}$$

If Newton's third law applies, the first (double) sum vanishes, and we get Newton's law for the total momentum in terms of the total external force $\vec{F}^{(e)} = \sum_i \vec{F}_i^{(e)}$

$$\dot{\vec{P}} = \vec{F}^{(e)}. \tag{14}$$

For zero external force

$$\dot{\vec{P}} = 0, \tag{15}$$

and the total momentum is conserved. We can also write the total momentum as

$$\vec{P} = M\frac{d\vec{R}}{dt} \tag{16}$$

with $M = \sum_i m_i$ the total mass, and $\vec{R} = \sum_i m_i \vec{r_i}/M$ the *center of mass* coordinate. Equation (14) for the case when Newton's third law is true, can be written

$$M\frac{d^2\vec{R}}{dt^2} = \vec{F}^{(e)}. \tag{17}$$

These equations show that if Newton's third law holds, the internal forces cancel, and we can write the same laws of motion for the center of mass coordinate of a composite body.

For a set of particles with vector positions $\vec{r_i}$ measured from origin O, the *angular momentum* about O is defined as

$$\vec{L} = \sum_i \vec{r_i} \times \vec{p_i}. \tag{18}$$

Taking the time derivative

$$\dot{\vec{L}} = \sum_i \dot{\vec{r_i}} \times \vec{p_i} + \sum_i \vec{r_i} \times \dot{\vec{p_i}}. \tag{19}$$

The first term is zero, since $\vec{p_i} = m_i\dot{\vec{r_i}}$ is parallel to $\dot{\vec{r_i}}$. Now using Newton's second law

$$\dot{\vec{L}} = \sum_i \vec{r_i} \times \vec{F}_i^{(e)} + \sum_{ij} \vec{r_i} \times \vec{F}_{ji}. \tag{20}$$

The terms in the last sum can be paired up, such as $\vec{r_1} \times \vec{F}_{21} + \vec{r_2} \times \vec{F}_{12} = (\vec{r_1} - \vec{r_2}) \times \vec{F}_{21}$, where Newton's third law is used in the last equality. For the special case of *central forces*, the force $\vec{F}_{21}$ is along the vector separation of the particles $\vec{r_1} - \vec{r_2}$ and so the last term in Eq. (20) vanishes. This gives the equation of motion of the total angular momentum

$$\dot{\vec{L}} = \sum_i \vec{r_i} \times \vec{F}_i^{(e)} \equiv \vec{N}^{(e)} \quad \text{(central interparticle forces)}, \tag{21}$$

with $\vec{N}^{(e)}$ the total external *torque*. If the external torque is zero, angular momentum is conserved. Note that these results for the angular momentum were derived with the rather restrictive assumption of central interparticle forces. As we will see, they apply for rigid bodies more generally.

The state of a *rigid body* (e.g., a table) is defined by its center of mass coordinate and its orientation — six coordinates. The dynamics of the rigid body can be obtained from the total momentum equation Eq. (14) and the total angular momentum equation Eq. (21) — six equations. Rigid body dynamics can be quite complicated and interesting, and we will return to this in later lectures.

There is no such thing as a truly rigid body. There will always be some *elasticity* and *dissipation*. We can ignore it at first, then consider the more complex dynamics, in (thermal and/or mechanical) equilibrium, as a correction to the motion of the center of mass and orientation.

More to come soon, on:

- Newton's laws of motion
- Symmetries and Conservation laws
  - space translation invariance (homogeneity) $\Rightarrow$ conservation of momentum
  - rotational invariance (isotropy) $\Rightarrow$ conservation of angular momentum
  - time translation invariance $\Rightarrow$ conservation of energy
- Transformation laws:
  - Rotational $\Rightarrow$ vectors
  - All inertial frames equivalent $\Rightarrow$ Galilean invariance