Physics 106a: Assignment 6

15 November, 2019; due 7pm Friday, 22 November in the “Ph106 In Box” in East Bridge mailbox.

Scattering, and Hamiltonian Formulation

Reading

Scattering: Hand and Finch Chapter 4, especially 4.7 and problems 28-30.
Hamiltonian mechanics: Hand and Finch Chapter 5.

Problems

1. **Cross section for Rutherford scattering** (H& F problem 4.28):

   We sketched out the derivation of the Rutherford scattering cross section in class; here, you will fill out all the steps in detail. (Other textbooks go through the calculation in detail, and it's ok if you had read about it earlier). A parallel beam of energetic alpha particles (helium nuclei from radium decay) of kinetic energy E is sent towards a thin gold foil, scattering off of individual gold nuclei.

   (a) Assume that the potential is that of a “point-like” scatterer, so that $v(r) = Z_{Au} Z_\alpha e^2/r$ down to the smallest values of r accessible by the experimental conditions. Starting with the formula relating the impact parameter b to the scattering angle θ, derive the differential cross section for Rutherford scattering:

   $$\frac{d\sigma}{d\Omega} = \left(\frac{Z_{Au} Z_\alpha e^2}{4E}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

   You can assume that the gold nucleus target is so much heavier than the beam alpha particle that it is at rest in the center of mass (neglect recoil), and the total center-of-mass energy is equal to the kinetic energy of the beam alpha particle, E.

   (b) Sketch / plot the differential cross section as a function of θ.

   (c) How close (minimum distance of approach) will an alpha particle with kinetic energy E come to the gold nucleus, in terms of all of the parameters of the problem?

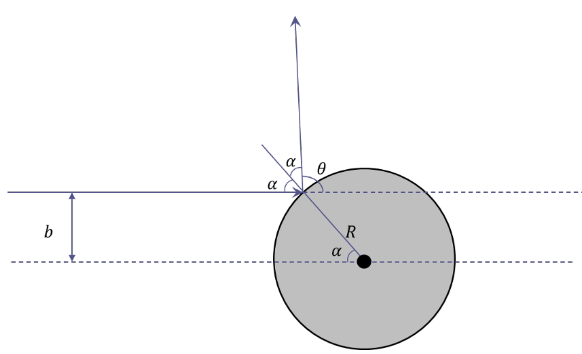
   (d) Numerically, how close (minimum distance of approach) will an alpha particle with 5.3 MeV kinetic energy come to the gold nucleus? You may use the following numerical values: $Z_{Au} = 79$ and $Z_\alpha = 2$, the classical electron radius $r_c = 2.817 \times 10^{-13}$ cm (a convenient way to express e^2), $m_e c^2 = 0.511$ MeV, $m_\alpha c^2 = 3728$ MeV, $m_{Au} c^2 = 183,471$ MeV.

   (e) Would the scattering cross section be different if the potential were attractive instead of repulsive?

   (f) Qualitatively, what would you expect will happen to the angular distribution of scattering if the beam particles are sufficiently energetic to penetrate inside the nucleus? What minimum energy would you need to accomplish this, if The radius of a gold nucleus is approximately 7.5 fm = 7.5×10^{-13} cm.

2. **Cross section for hard sphere scattering**

   A point projectile scatters off of a target rigid sphere of radius R, as depicted below from the side.



(a) Compute the differential scattering cross section. Integrate your result over all scattering angles to find the total cross section.

(b) Instead, assume that the projectile has a radius of R_b and the target has a radius of R_t. Again, compute the differential and total cross-sections.

3. **Lab (fixed target) frame and center-of-mass frame**:

   In the class lecture notes, review the computations that lead to relations between the scattering angle of the beam particle (of mass m) in the lab frame (where the heavier target particle, of mass M, starts out at rest) and in the center of mass frame:

$$\tan\theta_{\text{lab}} = \frac{\sin\theta_{\text{cm}}}{\cos\theta_{\text{cm}} + m/M},$$

   and of the resulting cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}} = \frac{\sin\theta_{\text{cm}}}{\sin\theta_{\text{lab}}}\frac{d\theta_{\text{cm}}}{d\theta_{\text{lab}}}\left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}}.$$

   (a) Show that for equal mass particles, $\theta_{\text{lab}} = \theta_{\text{cm}}/2$.

   (b) Show that for equal mass particles,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}} = 4\cos\theta_{\text{lab}}\left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}}.$$

   (c) For two hard spheres (as in the previous problem) of equal mass, verify that the total cross section in the lab frame is $\pi(R_b + R_t)^2$.

4. **Hamiltonian treatment of nonlinear oscillator**: The Hamiltonian of a 1 degree of freedom system is

$$H = \tfrac{1}{2}p^2 - \tfrac{1}{2}q^2 + \tfrac{1}{4}q^4.$$

   (a) Find the equations of motion $\dot{q}, \dot{p}$

   (b) Find the equilibria, and calculate the frequency of small oscillations about any stable equilibria.

   (c) Sketch a variety of trajectories of the dynamics in phase space (q, p) (including the direction they are traversed) and indicate any equilibria. Make sure you include enough trajectories to illustrate: small amplitude oscillations about stable equilibria; the behavior near any unstable equilibria; large amplitude oscillations; and the homoclinic or heteroclinic orbit(s) (i.e. the infinite period orbit(s) starting and returning to an unstable equilibrium).