LIGO antenna pattern (without knowing any GR).
(a simple pedagogical note by AJW, 4/01/01)

ETM response to quadrupole wave must be of the form: $dLx = x^i T^{ij} x^j$
where $x^i$ is displacement from ITM to ETM (take to be unit x, y vectors with length $L_{\text{arm}}$)
and $T^{ij}$ in the "transverse traceless" gauge must be built out of the vectors characterizing the GW:
the direction of motion $w^i$, and the direction of the GW strain perpendicular to the direction
(transverse) and each other, $u^i$ and $v^i$.
It must be traceless and anti-symmetric under $u \leftrightarrow v$, so it must be of the form:
$T^{ij} = u^i u^j - v^i v^j + a (u^i v^j - v^i u^j)$
where I don’t know what $a$ is, but it doesn’t matter, since that term is zero when $x^i T^{ij} x^j$ is
formed. This gives, for $x^i T^{ij} x^j / L^2$, in the $(Lx-Ly)/2$ combination:

Generated with the following code:

```matlab
% construct GW direction and x/y polarization vectors
hz = [-sinth*cos(phi) -sinth*sin(phi) -costh];
wx = [-hz(2) hz(1) 0]./sqrt(hz(1)^2+hz(2)^2);
wy = cross(hz,wx);
hx = cos(psi)*wx+sin(psi)*wy;
hy = cross(hz,hx);

% convert to Lx and Ly with the antenna pattern
Lx = (hx(1)^2-hy(1)^2);
Ly = (hx(2)^2-hy(2)^2);
Lm = 0.5.*(Lx-Ly);
Lp = 0.5.*(Lx+Ly);
```
cLc(I,J) = Lm;

These patterns agree perfectly with the usual formulae for the antenna pattern:

% analytic form of antenna pattern
FLm = 0.5*(1+costh^2)*cos(2*phi)*cos(2*psi)  
     -costh*sin(2*phi)*sin(2*psi);
FLp = sinth^2*cos(2*psi);